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The Emergent String Conjecture

Seung-Joo Lee, Wolfgang Lerche, Timo Weigand

Abstract

We present a brief and elementary overview of the Emergent String Conjecture and recent developments.

Contents

1 Introduction

One of the fundamental open problems in theoretical physics is the question: "Which types of quantum gravity (QG) are possible?" String Theory, at low energies leading to gravity coupled to particle physics, ought to provide a canonical answer. This provides a "top down" approach where one starts with an underlying, arguably consistent theory and determines what kinds of effective field theories (EFT) it can produce. It has turned out that the number of such consistent effective theories is huge, given the enormous number of ways to compactify ten dimensional string theory down to four dimensions. This makes this approach of little practical use, and one may also object that since starting from a consistent string theory, it is no surprise that one ends up with a consistent theory. Thus, this "seeking under the lamp-post" strategy may preclude other potential theories of quantum gravity unrelated to string theory.

This is why, starting from the work [Vaf05], there was a shift of focus to a "bottom up" strategy which revolves around the question "what are general constraints on ordinary quantum field theories that admit a consistent UV completion and can be coupled to gravity?" The set of such consistent theories has been dubbed "Landscape", while the rest lies in the "Swampland". The point here is not to look for naive field theoretical properties like renormalizability or absence of gauge and gravitational anomalies, but rather for much more subtle and general constraints that are invisible in ordinary quantum field theory, for example requiring consistent decay modes of black holes [AHMNV06].

A number of important results have been obtained in recent years, mostly inspired by string theory but then abstracted thereof and conjectured to be more generally valid. While the individual statements are often not that strong, they form a web of interconnected consistency conditions whose combined significance is far more than of the sum of its parts. It is this multi-faceted structure what gives strength to the Swampland Program as a whole (see [BCV17, Pal19, vBCIMV21, GnH21, ABKV22] for reviews).

One important such condition on quantum gravity theories is the absence of global symmetries (which, if present, would be in conflict with a statistical interpretation of black hole entropy) [BS10]. Other, related conditions constrain the spectrum of massive states, beyond those required by the low energy EFT by its own field theoretical consistency. A particular example is the *Weak Gravity Conjecture* [AHMNV06] (WGC, see [Pal20, HHRR22] for reviews), which gives definite constraints on the spectrum of massive states.

An important handle on the massive spectrum exists in theories that have a moduli space, $\mathcal{M}$, of vacua. This applies in particular to theories with extended supersymmetry, and partially also to compactifications with $N = 1$ supersymmetry in four dimensions. Here the massive spectrum changes when we move in the moduli space, ie., when changing vacuum expectation values of massless scalar fields. It has turned out that infinite distance limits are particularly important, which can be thought of as cusp-like boundary regions of $\mathcal{M}$. In these regimes, an infinite tower of massive states becomes parametrically light, which is the content of the *Swampland Distance Conjecture* [OV06]. More concretely, considering two points p and q in the moduli space whose distance d_{pq} in its natural metric becomes large, an infinite tower of states must become light with characteristic scale

$$M_q \;=\; M_p \exp\left(-\alpha \frac{d_{pq}}{M_{\mathrm{Pl}}}\right) \;\longrightarrow\; 0\,. \tag{1.1}$$

Here M_{Pl} is the Planck mass and the exponential vanishing rate is parametrized by $\alpha > 0$. The physical motivation is that the EFT must break down in this regime: In many cases, some coupling constants, e.g., of gauge fields or strings, vanish there, which reduces a local gauge symmetry to a global one, which in turn would violate the "no global symmetry" constraint of quantum gravity.

A significant property is that in such limits the asymptotically massless states are weakly coupled, when represented in a properly adapted duality frame. It is important to understand the nature of asymptotically massless towers and the possible perturbative quantum gravities that can arise in these regimes near the boundary of $\mathcal{M}$. Are there new, hitherto unknown theories to be discovered, analogous to the non-perturbative, albeit non-gravitational strings [Wit95] that are known to arise at certain singularities in the interior of the moduli space? Or are certain limits pathological and lead to ill-defined theories?

By investigating such limits in detail from a top-down perspective, a simple and presumably universal picture has emerged, which is phrased in terms of the *Emergent String Conjecture* (ESC) [LLW19c]. This is the topic of this brief review (for a more detailed variant see [Lee21]). The ESC was initially motivated by the possible asymptotic geometries of the boundaries of $\mathcal{M}$, which for certain cases had been classified in the mathematical literature. Upon closer look, the precise way pathologies are avoided turns out to be surprisingly non-trivial. In fact, certain mathematical theorems seem always to conspire in precisely the right way in order that just sensible physics emerges and pathologies are avoided. Clearly this is not unexpected when starting from a consistent theory in the first place, but the principles of how this works serve as a guideline to address consistency requirements also from a more general bottom-up perspective. Indeed, as we will review, the ESC poses rather non-trivial constraints on effective field theories and has far-reaching consequences.

2 The Emergent String Conjecture – ESC

Before we go through these points in more detail, let us present the main conjecture. For this we make the following assumptions. First, we consider a quantum theory of Einstein gravity in asympotically flat $d \geq 4$ dimensions.[1] We also require the existence of a moduli space of vacua, $\mathcal{M}$. Then we formulate the Emergent String Conjecture as follows:

> Any infinite distance limit in $\mathcal{M}$ leads to precisely one of just two possibilities [LLW19c]:
>
> **(I) Emergent String Limit**: A *unique* critical string becomes asymptotically tensionless with respect to the Planck scale, as well as weakly coupled. The leading tower consists of string excitations, and if the spacetime dimension $d < 10$, Kaluza-Klein (KK) modes are found at the same scale, reflecting compactification of the critical string to $4 \leq d < 10$ dimensions.
>
> **(II) Decompactification Limit**: One or several dimensions decompactify. The leading tower consists of KK modes only.

Note that strings can also appear in theories like M-Theory even though those are not fundamental theories of strings; rather strings emerge here as parametrically tensionless solitons from wrapped branes *as long as we do not decouple gravity.* They appear as weakly coupled critical strings only after a duality transformation. A non-trivial point is that no other, potentially pathological limits like novel string, multi-string or membrane limits can supposedly appear.

2.1 Initial Evidence

We now provide evidence for the ESC from the point of view of compactification geometry, ie., from the top-down perspective. To illustrate all non-trivial aspects

[1] While most of the initial evidence for the ESC concerned a quantum gravity theory with vanishing cosmological constant $\Lambda = 0$, we believe that the conjecture still holds with $\Lambda \neq 0$ as long as one restricts to Einstein gravity; see [BCI20, PRVV20, BCI23] for some background.

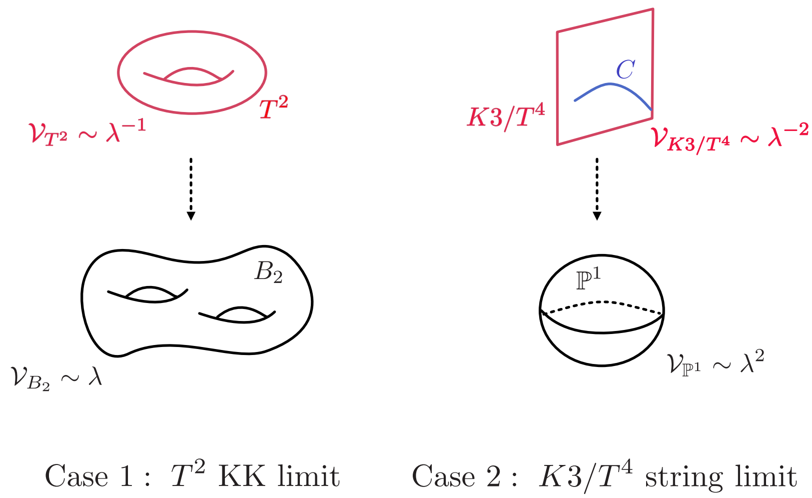

Figure 1: Infinite distance limits in the Kähler moduli space of Calabi-Yau threefolds with constant overall volume are necessarily fibrations with either a shrinking T^2 fiber or a shrinking $K3$ resp. T^4 fiber. These limits require properly co-scaled volumes of bases and fibers, as indicated.

of the ESC in a controlled setup, we consider a quantum gravity theory that is not already a weakly coupled string theory and focus, for definiteness, on M-Theory compactified on a Calabi-Yau threefold X_3. This yields an EFT with $N = 1$ supersymmetry in $d = 5$ dimensions. The Kähler moduli space $\mathcal{M}_K$ is parametrized by the Kähler form $J = t^a J_a$, $a = 1, ..., h^{1,1}(X_3)$, where J_a are the generators of the Kähler cone in some given basis. The Kähler moduli are flat coordinates on $\mathcal{M}_K$ and are given by the volumes $t^a = \int_{C^a} J$ of curves $C^a \in H_2(X_3, \mathbb{Z})$. Infinite distance limits then amount to sending one or more of those volumes to infinity, $t^a \to \infty$. The important point here is that in order not to decouple gravity, the Planck mass and thus the total volume $\mathcal{V}_{X_3} \sim \int J^{\wedge 3} \sim \mathcal{O}(t^3)$ must stay finite in this limit.[2] This implies that some of the remaining moduli must go to zero with an appropriate rate. Such limits are therefore characterized by highly anisotropic degenerating geometries. The physical consequence is that the $M2$ or $M5$ branes wrapped around the vanishing cycles lead to asymptotically massless states. This simple observation is the basis of the ESC.

The question arises about the nature of these towers of massless states, and about what possible quantum gravity theories govern the asymptotic weak-coupling regimes. This is where certain mathematical theorems, in particular by Oguiso [Ogu93], come into play. It turns out that to admit a finite-volume infinite distance limit in the Kähler moduli space $\mathcal{M}_K$, the threefold X_3 must necessarily

[2] Alternatively, if $\mathcal{V}_{X_3} \to \infty$, this may easily correspond to a decompactification limit in an obvious way, and after all, one can always rescale J such as to focus on the non-trivial finite-volume infinite distance limits [LLW19c].

admit the structure of a fibration such that in the limit the generic fiber shrinks (while the base of the fibration becomes large). In fact there are essentially only two possibilities for this, corresponding to fibers of complex dimension one (type T^2) or two (type $K3$ or T^4). We depict the situation in Figure 1, where we also indicate the relative scaling of the fibers and bases in terms of a parameter λ that asymptotes to infinity.

More specifically, for fiber type T^2 the leading tower is formed by $M2$-branes wrapping the fiber n times, which yields masses

$$M_n \;=\; nM_0\,, \qquad M_0 \;=\; T_{M2}\mathcal{V}_{T^2} \;\sim\; \lambda^{-1} \;\to\; 0\,. \tag{2.1}$$

This spectrum is typical for Kaluza-Klein modes and indeed corresponds, via duality, to decompactification from 5 to 6 dimensions (essentially, from M- to F-theory).

The other situation is more interesting: here an $M5$ brane wraps the $K3$ or T^4 fiber. This yields an "emergent" solitonic string in 5 dimensions [HS95], which is dual to a perturbative, critical heterotic or Type II string, respectively, with tension

$$T_{\rm str} \;=\; T_{M5}\mathcal{V}_{K3/T^4} \;\sim\; \lambda^{-2} \;\to\; 0\,. \tag{2.2}$$

Since the spacetime dimension of the effective field theory does not change in this case, we call such string limits also equidimensional limits. Furthermore, there are two KK-like towers whose characteristic masses scale with the same rate:

$$M^2_{\rm KK;1} \;=\; {\mathcal{V}_{\mathbb{P}^1}}^{-1} \;\sim\; M^2_{\rm KK;2} \;=\; {\mathcal{V}_C}^{2} \;\sim\; \lambda^{-2} \;\to\; 0\,. \tag{2.3}$$

The first arises from KK modes from the large $\mathbb{P}^1$ base, while the second arises from $M2$ branes wrapping curves C with $C\cdot_{K3} C \geq 0$ in the fibers (as sketched in Figure 1). In terms of the perturbative duality frame of the critical heterotic or Type II strings, these modes correspond to the KK towers from the compactification from 10 to 5 dimensions.

Let us note that while $M5$ branes on general surfaces do not yield critical strings (for example, MSW strings [MSW97]), critical heterotic and Type II strings do arise when they wrap $K3$ or T^4 surface fibers. That these are precisely the fiber geometries which shrink in large distance limits of $\mathcal{M}_K$ at constant volume and which lead to weakly coupled quantum gravity theories is another example of how physical consistency is anchored in geometry.

We can iterate on this by asking: Could two different critical strings become weakly coupled at the same rate, so that we potentially have a pathological situation with say, two gravitons? Such a geometry can be easily arranged by considering a threefold X_3, which is $K3$ and/or T^4 fibered in multiple ways, so that their respective generic fibers can be dialed to shrink at the same rate: e.g., $\mathcal{V}_{K3,i} \sim \lambda^{-2}$ for $i = 1, 2$. However one can show [LLW19c] that X_3 then necessarily admits a further compatible T^2 fibration, whose fiber shrinks at a faster rate $\mathcal{V}_{T^2} \sim \lambda^{-4}$, leading to

$$M_{\rm KK} \sim \mathcal{V}_{T^2} \sim \lambda^{-4} \ll \lambda^{-1} \sim \mathcal{V}^{1/2}_{K3,i} \sim M_{\rm str}\,. \tag{2.4}$$

Thus the dominant tower that sets the asymptotic duality frame is a KK tower and not an emergent string tower. This is precisely in line with the ESC, which posits that a leading tower is always unique. Furthermore, the indicated scaling is precisely such as to guarantee that the tension of the two strings sits at the Planck scale of the theory obtained by decompactification to six dimensions, which coincides with the species scale associated with the leading KK tower. Hence in the asymptotic theory the two strings are indeed strongly coupled and in particular do not set the duality frame so that no inconsistency with the potential appearance of two types of graviton can arise.

That $T_{\rm str} \sim M^2_{\rm KK}$ in the emergent string limit below 10 dimensions as in (2.2) and (2.3), ie., KK modes accompany a critical string tower, is of course expected, as for $T_{\rm str} \ll M^2_{\rm KK}$ new weakly coupled strings would appear which would not arise from compactification from higher dimensions. A priori, such situations could in principle occur, for example by considering a two-parameter limit involving an overall rescaling, $\mathcal{V}_{X_3} \to \mu^3 \mathcal{V}_{X_3}$ for $\mu \to 0$. Then at first sight one may arrange that indeed $M^2_{\rm str} \sim \mu^2\lambda^{-2} \ll M^2_{\rm KK} \sim \mu^{-1}\lambda^{-2}$. However, when $\mathcal{V}_{X_3} \to 0$ quantum corrections in the hypermultiplet moduli space become important which invalidate the argument. It can be shown in some explicit examples [LLW19c, BMW19, KLWW20, AGKW21] that these indeed shield the problematic regions in moduli space so that those are not accessible. This non-trivial manifestation of the ESC appears to protect consistency at a deeper level than naive degeneration geometry.

2.2 Further Evidence

Further support for the ESC has been accumulated by numerous studies of other geometries. Space permitting, we can present here only a brief, necessarily incomplete overview. A comprehensive study of the ESC in 9 dimensions was presented in refs. [CGnHPDF22, EHM$^+$23, EHR$^+$24]. Various large distance limits in the Kähler moduli space in 6, 5 and 4 dimensions (F/M/IIA-Theory on X_3) have been investigated in refs. [LLW18, LLW19b, LLW19c, Rud23]. In F-Theory, emergent strings arise from wrapping $D3$ branes on $\mathbb{P}^1$ fibers (which leads to heterotic strings) or T^2 fibers (which leads to Type II strings). F-Theory on fourfolds with $N = 1$ supersymmetry in 4 dimensions have been discussed in [LLW19a, KLWW20]. Here we meet again the phenomenon that quantum corrections preclude critical strings with tension less than the KK scale.

So far we have been concentrating on large distance limits at constant volume in the Kähler (vector multiplet) moduli space, whose asymptotic fibration geometry is governed by Oguiso's Theorem [Ogu93]. The complex structure side of the story is much less understood, partly because in the large distance limits in $\mathcal{M}_{CS}$ the degenerating geometry is less intuitive. Rather, as has been shown in refs. [GPV18, GLP18, GRvdH19], the relevant asymptotic mixed Hodge theory of Calabi-Yau n-folds is characterized by monodromy orbits.[3]

While the evidence discussed so far exclusively concerns the moduli space of the internal bulk geometries, in the context of F-theory the large distance limits

[3]The asymptotic limits of $\mathcal{M}_{CS}$ can be mapped to $\mathcal{M}_K$ via mirror symmetry [CGV18]. However, this exchanges Type IIA and Type IIB strings, so we stay in the vector multiplet moduli space. The more complicated structure we talk about here refers to the hypermultiplet moduli space of the same given Type II string.

in $\mathcal{M}_{CS}$ include those that correspond to the extremal internal arrangements of 7-branes. They are qualitatively distinct from the limits of the bulk geometries and have initially been analyzed in 8 dimensions, for F-Theory on elliptic $K3$ [LW21, LLW21, CGnHPDF22].

The relevant asymptotic geometries involve the fibral singularities of non-Kodaira type, which are not as well understood as the well-known Kodaira singularities found in the interior of the moduli space. In general such a limiting geometry decomposes into a union of several components associated with a log-Calabi Yau structure. For $K3$ the situation is well tractable and one finds that there are in total four types of degenerations, refining the classification of the so-called Kulikov models [Kul77, Kul81]. These can be perfectly matched either to decompactification to 9 and 10 dimensions, or to an emergent Type II string.

While the situation is more complicated for less than 8 dimensions, the ESC for the (hypermultiplet) moduli space $\mathcal{M}_{CS}$ has been supported by studying numerous examples. This includes in 6 dimensions F-Theory on elliptic X_3 still concerning the 7-brane moduli limits [AGLW23], in 4 dimensions Type II strings on X_3 [MM22, BMW19] and F-Theory on elliptic X_4 [vdH24]. As expected, all results are in line with the ESC.

For more general geometries, the ESC for M-Theory on manifolds with G_2 holonomy was discussed in [Xu20], for mixed Kähler-complex structure limits for Type IIA strings on T^6 in [BGP23], and for non-geometric backgrounds in [ABL24]. Non-supersymmetric theories were considered in [Bas22].

Other, more broad discussions orbiting around the weak gravity conjecture, large distance limits and the ESC include ref. [EHK$^+$22] which puts a lower bound on the exponential decay rate in eq. (1.1): it was argued that generally $\alpha \geq \frac{1}{\sqrt{d-2}}$ which is saturated precisely by emergent string towers. Relatedly, extensive evidence for a universal pattern of scaling laws for various infinite distance limits was presented in [CRV23a, CRV23b, CICHIn23]. The latter's findings suggest that the ESC can be derived also from a bottom-up perspective, ie., from an EFT point of view without specific reference to an UV completion.

Indeed, fully in the spirit of the original landscape paradigm of quantum gravity, refs. [BLM23, BMW24] translate constraints from the thermodynamics of black holes into the appearance of towers of parametrically light excitations. These are intrinsically tied to black holes of minimal size, which may be seen as a manifestation of UV/IR mixing that is inherent to quantum gravity. Intriguingly, these towers must be either KK towers or towers with exponentially growing degeneracies. While the latter are characteristic for string excitations, there is yet no firm proof that these actually do belong to weakly coupled critical strings. Nevertheless these results provide non-trivial evidence for the ESC from a bottom-up perspective, without explicitly relying on string theory or even supersymmetry.

3 Consequences of the ESC

The ESC has a number of rather strong implications for consistent quantum gravity theories.

A first important consequence of the ESC is that there are no tensionless membrane (or higher-dimensional brane) limits, for which the characteristic scale of some hypothetical "critical" membrane would become dominant [AGKW21]. Here what we mean by a "critical" membrane is that its S^1 reduction would produce a critical string. This claim is a direct consequence of the ESC via dimensional reduction because a putative emergent membrane limit would map to a pathological string limit for which $T_{\rm str} < M_{\rm KK}^2$ under circle reduction, which according to the ESC is not allowed. More concrete quantitative checks involve matching the hypermultiplet moduli spaces $\mathcal{M}_{CS}$ of M-Theory on $X_3 \times S^1$, of Type IIA strings on X_3 and of Type IIB strings on the mirror $\hat{X}_3$, and emergent string limits are indeed found to be obstructed by quantum corrections.

Second, as already mentioned, the ESC provides [EHK$^+$22] a lower bound $\alpha \geq \frac{1}{\sqrt{d-2}}$ for the exponential decay rate α appearing in the Distance Conjecture in eq. (1.1). The existence of such a bound (which is saturated by emergent string towers) is particularly important for all concrete applications of the Distance Conjecture e.g. to constrain cosmological scenarios.

The ESC also *explains* the appearance of a super-extremal tower of particle states for asymptotically weakly coupled gauge theories coupled to gravity. First, it can be used to argue [FCMWW23] that in all such weakly coupled limits, consistency of the WGC under circle reduction indeed requires the appearance of a super-extremal tower of states along the lines of [HRR15]. Second, since weakly coupled theories are either decompactification or emergent string limits according to the ESC, super-extremal towers are guaranteed to exist because these can be found explicitly within the respective spectra of asymptotically light particles in both classes of theories.

A particularly spectacular, but also more speculative application of the ESC arises in the Dark Dimension scenario proposed in [MVV22]: The starting point is the idea that the smallness of the cosmological constant suggests that the quantum gravity describing our Universe should arise in a (generalized) infinite distance limit of a family of theories with increasingly small cosmological constant. The AdS Distance Conjecture of [LPV19], taken at face value also for positive values of the cosmological constant, then implies the appearance of a tower of states whose mass decreases with Λ^{-c} for some value of $\frac{1}{4} \leq c \leq \frac{1}{2}$. According to the ESC, such limits are either decompactification or emergent string limits, but the latter possibility is phenomenologically ruled out as it would, in the present case, imply a string scale that is too low to not have been observed already at the LHC. The only remaining option consistent with current experimental and observational bounds is the decompactification of a single mesoscopic dimension [MVV22].

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Center for Theoretical Physics of the Universe
Institute for Basic Science, Daejeon 34126, South Korea.
E-mail address: seungjoodotlee at gmail.com

Theory Department, CERN, Geneva, Switzerland.
E-mail address: wolfgang.lerche at cern.ch

II. Institut für Theoretische Physik
Universität Hamburg,
Luruper Chaussee 149, 22761 Hamburg, Germany and
Zentrum für Mathematische Physik
Universität Hamburg, Bundesstrasse 55, 20146 Hamburg, Germany.
E-mail address: timo.weigand at desy.de